Eq. separáveis

Uma EDO de 1ª ordem é <u>separável</u> se pode ser escrita como

$$y(x) = ?$$

$$y'(x) = \frac{dy}{dx}(x)$$

$$f(y)\frac{dy}{dx} = g(x)$$

ou

$$f(y)\,y' = g(x).$$

Resolvendo:

mudança de var.

$$f(y)\frac{dy}{dx} = g(x) \Rightarrow \int f(y)\,dy = \int g(x)\,dx$$

Exemplos: 1) $\dfrac{dy}{dx} = \dfrac{x^2}{y^2}$ $\qquad y' = \dfrac{x^2}{y^2}$

$(\times y^2)$

$$\Rightarrow \underbrace{y^2}_{f(y)}\frac{dy}{dx} = \underbrace{x^2}_{g(x)}$$ é separável.

$$\therefore \int y^2 dy = \int x^2 dx \Rightarrow \frac{y^3}{3} + c_1 = \frac{x^3}{3} + c_2$$

$$\Rightarrow \frac{y^3}{3} = \frac{x^3}{3} + c_2 - c_1 \overset{(\times 3)}{\Rightarrow} y^3 = x^3 + \underbrace{3c_2 - 3c_1}_{c} \Rightarrow y = \sqrt[3]{x^3 + c}$$

Verificando:

$$y = (x^3+c)^{1/3} \Rightarrow y' = \frac{1}{\cancel{3}}(x^3+c)^{-2/3}\cdot \cancel{3}x^2 = \frac{x^2}{(x^3+c)^{2/3}} = \frac{x^2}{(\underbrace{\sqrt[3]{x^3+c}}_{y})^2}$$

$$= \frac{x^2}{y^2}$$

$y = \sqrt[3]{x^3 + C}$ são soluções da EDO.

PVI: $y(0) = 2$

$$2 = y(0) = \sqrt[3]{0^3 + C} \Rightarrow \sqrt[3]{C} = 2 \Rightarrow C = 8.$$

$$\therefore\ y = \sqrt[3]{x^3 + 8}$$

$$3x + 2 = 5$$

$$3x + 2 - 2 = 5 - 2$$

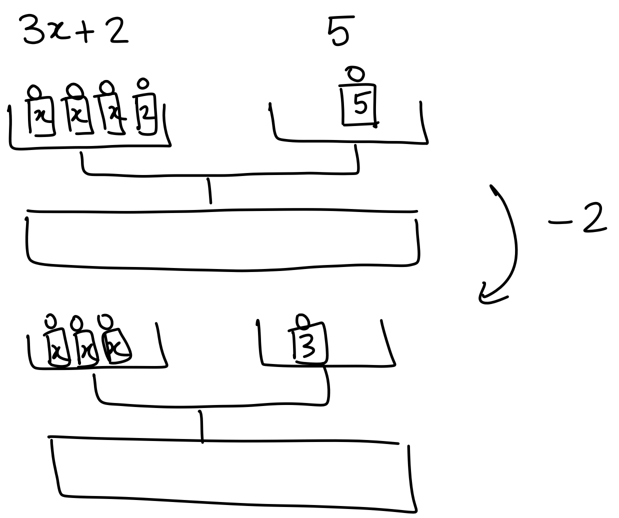
$$3x = 5 - 2$$

$$3x = 3$$

$$\frac{3x}{3} = \frac{3}{3}$$

$$x = \frac{3}{3}$$

$$x = 1$$



2) $$y' = \frac{6x^2}{2y + \cos y} \Rightarrow (2y + \cos y)\frac{dy}{dx} = 6x^2$$

$$\Rightarrow \int 2y + \cos y\, dy = \int 6x^2 dx \Rightarrow y^2 + \operatorname{sen} y + C_1 = 2x^3 + C_2$$

$$\Rightarrow y^2 + \operatorname{sen} y = 2x^3 + C$$ (eq. implícita)
(sol. implícita)

$$y^2 - 2y = x$$

$$y^2 - 2y + 1 - 1 = x \Rightarrow \underbrace{y^2 - 2y + 1}_{(y-1)^2} = x + 1 \Rightarrow (y-1)^2 = x + 1$$

$$\Rightarrow y = \pm\sqrt{x+1} + 1$$

3) $y' = x^2 y \underset{(y \neq 0)}{\overset{(\div y)}{\Longrightarrow}} \frac{1}{y} \cdot y' = x^2 \Rightarrow \int \frac{1}{y}\, dy = \int x^2 dx$

$\Rightarrow \ln|y| = \frac{x^3}{3} + C \Rightarrow e^{\ln|y|} = e^{\frac{x^3}{3}+c}$

$\Rightarrow |y| = e^{\frac{x^3}{3}+c} \Rightarrow y = \begin{cases} e^{\frac{x^3}{3}+c}, & y \geq 0 \\ -e^{\frac{x^3}{3}+c}, & y < 0 \end{cases}$

$\Rightarrow y = \begin{cases} e^{\frac{x^3}{3}} \cdot \overset{k>0}{e^{c}}, & y \geq 0 \\ -e^{\frac{x^3}{3}} \cdot \overset{k>0}{e^{c}}, & y < 0 \end{cases} \Rightarrow y = k e^{\frac{x^3}{3}}, \quad k \in \mathbb{R}^*$

Observe que $y(x)=0$ também é solução.

4) $y'' = x$

Chame $u = y' \Rightarrow u' = y''$:

$y'' = x \Rightarrow u' = x \Rightarrow \int 1\, du = \int x\, dx \Rightarrow u = \frac{x^2}{2} + c_1$

$\therefore y' = \frac{x^2}{2} + c_1 \Rightarrow \int 1\, dy = \int \frac{x^2}{2} + c_1\, dx \Rightarrow y = \frac{x^3}{6} + c_1 x + C_2$

$x^2 = x \overset{\div x}{\underset{x \neq 0}{\Rightarrow}} \frac{x^{\cancel{2}}}{\cancel{x}} = 1 \Rightarrow \underline{x = 1}.$

$0^2 = 0$

$x^2 - x = 0 \qquad \Delta = \ldots$

$x(x-1) = 0$